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THE ROYAL SOCIETY

Survival analysis of fishing vessels rolling in rough seas

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A new approach to the problem of predicting the safety of vessels rolling in rough seas is described. It is based on the state of the art in nonlinear dynamics of a single-degree-of-freedom system. The random wave excitation depends on sea state, vessel speed and direction of wave propagation. The differential equation of rolling motion is integrated by the harmonic acceleration method. The procedure is illustrated for the case of a typical fishing vessel. The roll response of an intact and damaged vessel is presented in the time and frequency domain. The fractal erosion of the safe basin in the initial-value plane is analysed. Finally, these results are used to determine the probability of a vessel's survival as a function of sea state, vessel speed and heading angle.

Keywords: fishing vessel; irregular waves; nonlinear rolling; roll energy spectrum; safe basin; probability of survival

1. Introduction

Vessels are designed in accordance with national and/or international rules. The stability criteria prescribe the form and area under the righting-arm curve based on experience (SNAME 1988). This is a rather conservative way to preserve a vessel from capsizing, since a very important role of nonlinear dynamic phenomena is not taken into consideration (Cartmell 1990; Dimentberg 1988; Thompson 1993). The fact that many intact vessels were lost in rough seas confirms this statement (Dahle & Nisja 1984). Therefore, the state of the art in nonlinear dynamics should be used in stability analysis, and consequently new stability criteria should be established.

A vessel rolling in regular beam seas has been quite well investigated, mostly in the frequency domain, using relevant numerical methods (Cardo *et al*. 1984). Nonlinear phenomena, such as amplitude jumping, superharmonic and subharmonic response, symmetry breaking and period doubling, have been analysed (Senjanović 1994; Peyton Jones & Cankaya 1996). Transition from regular to chaotic motion is considered in the time domain (Senjanović & Fan 1994). It is an indication of possible vessel capsize (Thompson *et al*. 1990). Chaotic rolling is partly and fully developed in the case of an intact and damaged vessel, respectively (Kan & Taguchi 1991).

In reality, a vessel rolls in rough seas, and the linear random process valid for small amplitude motion is solved by the spectral analysis (Price & Bishop 1974; Lloyd 1989; SNAME 1989). However, nonlinear response may be reliably determined only by numerical simulation (Falzarano *et al*. 1995; Senjanović & Fan 1995*a*). The explicit nonlinear phenomena caused by regular wave excitation are coupled and

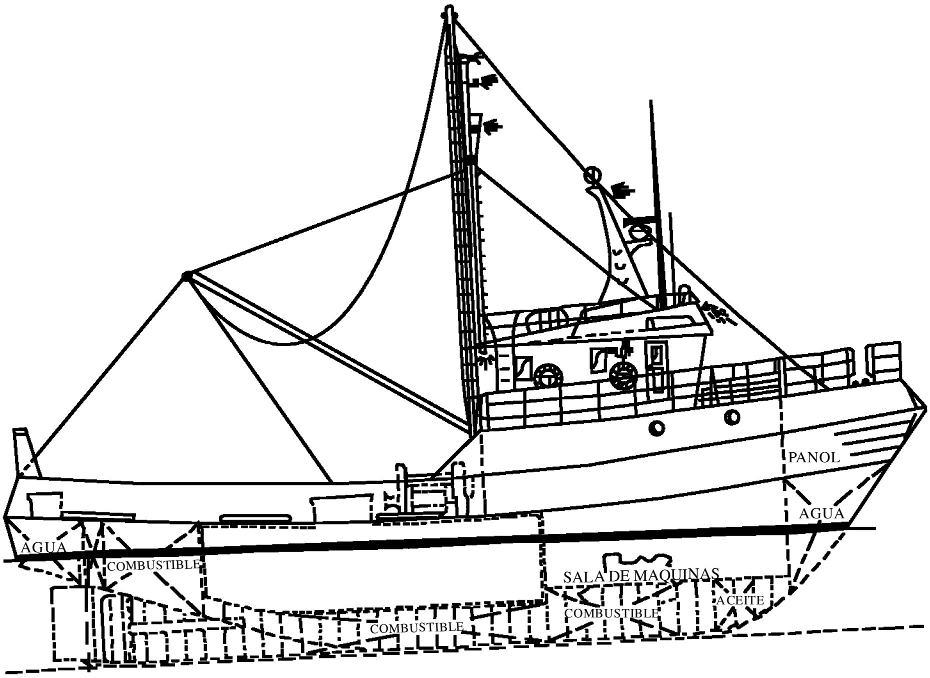

Figure 1. A typical fishing vessel.

interferated in the case of a spectral excitation (Ochi & Bolton 1973). The definition of deterministic chaos related to regular wave excitation can be extended to random chaos in the case of narrow band spectral excitation (Senjanović & Fan 1995*b*).

Taking the above facts into account, this article offers a new, sophisticated, dynamic approach to the problem of determining a vessel's safety in rough seas. Vessel rolling is simulated by a single-degree-of-freedom (SDOF) system. Nonlinear viscous damping and restoring moment are taken into account. Random wave excitation depends on the relevant wave slope energy spectrum for a given sea state and encounter frequency. The governing differential equation of rolling motion is solved in the time domain by the harmonic acceleration method. The safe basin in the initial-value plane is determined for different sea states and values of encounter frequency. The results obtained are used to predict the probability of a vessel's survival in rough seas as a function of sea state, vessel speed and heading angle. The procedure is illustrated for the case of a typical fishing vessel, like the one shown in figure 1, in both intact and damaged conditions.

2. Equation of rolling motion

A vessel in rough seas performs a complex motion consisting of six components: surge, sway, heave, roll, pitch, and yaw (Price & Bishop 1974; Lloyd 1989; SNAME 1989). Rolling motion is highly nonlinear. By neglecting coupling (Falzarano *et al.* 1995), for the sake of simplicity, the vessel rolling is analysed by an SDOF system. The governing differential equation of motion expresses the equilibrium of moments,

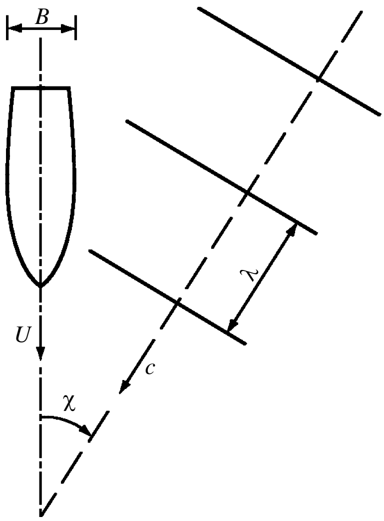

Figure 2. Definition of heading angle.

and it reads

$$I\ddot{\varphi} + D(\dot{\varphi}) + R(\varphi) = M(t), \tag{2.1}$$

where I is the virtual moment of inertia, D is the damping moment, R is the restoring moment, M is the wave-excitation moment and φ is the roll angle.

The virtual moment of inertia consists of the vessel's mass moment and the added mass moment of surrounding water, i.e.

$$I = I_{\mathrm{s}} + I_{\mathrm{w}}. \tag{2.2}$$

The damping moment, consisting of wave radiation and viscous components, may be approximated by a cubic polynomial as an analytical function

$$\begin{aligned} D(\dot{\varphi}) &= D_{\mathrm{w}}\dot{\varphi} + D_{\mathrm{v}}\dot{\varphi}|\dot{\varphi}| \\ &= D_1\dot{\varphi} + D_3\dot{\varphi}^3. \end{aligned} \tag{2.3}$$

The restoring moment is hydrostatic and is given by a nonlinear odd function. It may be represented by a seventh-order polynomial

$$R(\varphi) = K_1\varphi + K_3\varphi^3 + K_5\varphi^5 + K_7\varphi^7. \tag{2.4}$$

Substituting the above formulae (2.2), (2.3) and (2.4) into (2.1), and dividing the result by the virtual moment of inertia, the final form of the differential equation of motion is obtained:

$$\ddot{\varphi} + d_1\dot{\varphi} + d_3\dot{\varphi}^3 + k_1\varphi + k_3\varphi^3 + k_5\varphi^5 + k_7\varphi^7 = m(t), \tag{2.5}$$

where

$$\left.\begin{aligned} d_i &= D_i/I, \qquad i = 1, 3, \\ k_i &= K_i/I, \qquad i = 1, 3, 5, 7, \\ m(t) &= M(t)/I \end{aligned}\right\} \tag{2.6}$$

are relative roll parameters.

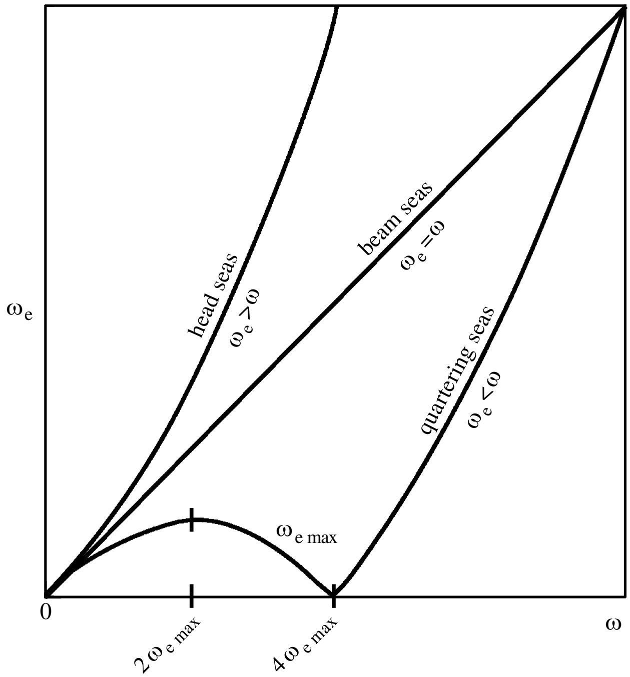

Figure 3. Encounter frequency as a function of wave frequency.

3. Wave excitation

The relative roll-excitation moment of a regular wave of beam sea may be presented in the following form (Contento *et al*. 1996*a*):

$$m(t) = \alpha_0 \omega_0^2 \pi \frac{h}{\lambda} \cos \omega t, \tag{3.1}$$

where α_0 is the effective wave slope coefficient (assumed constant), h is the wave height, λ is the wavelength of beam sea, ω_0 is the roll natural frequency, and ω is the wave frequency. Equation (3.1) may be extended, taking vessel speed U and heading angle χ into account. The heading angle is the angle between the vessel direction and the direction of wave propagation at a celerity c (see figure 2). Thus,

$$m(t) = \alpha_0 \omega_0^2 \pi \frac{h}{\lambda} \sin \chi \cos(\omega_e t), \tag{3.2}$$

where ω_e is the encounter frequency,

$$\omega_e = \omega - \frac{\omega^2 U}{g} \cos \chi. \tag{3.3}$$

The encounter frequency is single valued for beam seas and multi-valued for quartering seas (as shown in figure 3), where

$$\omega_{e\,max} = \frac{g}{4U \cos \chi} \tag{3.4}$$

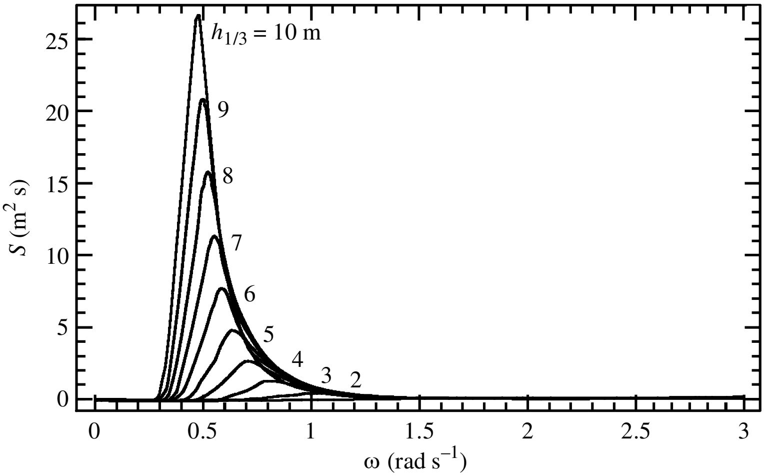

Figure 4. Tabain's wave energy spectrum of the Adriatic Sea.

and g is the acceleration due to gravity. The encounter frequency may be positive or negative. In the former case the waves overtake the vessel, and in the latter case the vessel overtakes the waves. If $\chi \to 90°$, then $\omega_{\mathrm{e}} \to \omega$ and $\omega_{\mathrm{e\,max}} \to \infty$.

Rough sea is presented by an irregular wave, i.e. as the sum of harmonic waves, so that the relative excitation moment (3.2) takes the form

$$m(t) = \alpha_0 \omega_0^2 \pi \sin\chi \sum_{n=1}^{N} \frac{h_n}{\lambda_n} \cos(\omega_{\mathrm{e}n} t + \varepsilon_n), \tag{3.5}$$

where n is the index of wave component and ϵ_n is the random phase angle in the range 0–2π, whose probability of occurrence, $1/(2\pi)$, is uniformly distributed.

The wave height is obtained from the wave energy spectrum $S(\omega)$ (Price & Bishop 1974; Lloyd 1989; SNAME 1989)

$$h_n = 2\sqrt{2\tilde{\omega} S(n\tilde{\omega})}, \tag{3.6}$$

where $\tilde{\omega}$ is the step of wave frequency, i.e. $\omega_n = n\tilde{\omega}$. The wavelength is given by

$$\lambda_n = \frac{2\pi g}{(n\tilde{\omega})^2}. \tag{3.7}$$

Furthermore, following (3.3), the encounter frequency for a wave component takes the form

$$\omega_{\mathrm{e}n} = n\tilde{\omega} - \frac{(n\tilde{\omega})^2 U}{g} \cos\chi. \tag{3.8}$$

The wave energy spectrum depends on ocean statistics (Price & Bishop 1974; Lloyd 1989; SNAME 1989). In this analysis, it is assumed that the vessel is exposed to the Adriatic Sea environment. For this closed sea, a specific wave energy spectrum (see figure 4) has been defined on the basis of observations and measurements (Tabain 1977),

$$S(\omega) = 0.862 \frac{0.0135 g^2}{\omega^5} \exp\left[-\frac{5.186}{\omega^4 h_{1/3}^2}\right] 1.63^p, \tag{3.9}$$

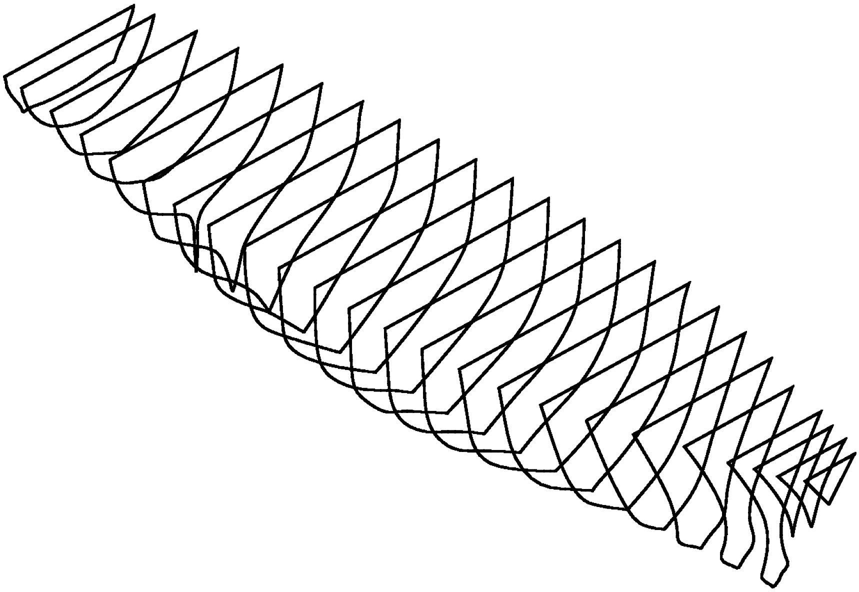

Figure 5. Hull form of the fishing vessel.

where $h_{1/3}$ is the significant wave height. The remaining quantities are defined as follows:

$$\left.\begin{aligned} p &= \exp\left[-\frac{(\omega-\omega_{\mathrm{m}})^2}{2\sigma^2\omega_{\mathrm{m}}^2}\right], \\ \omega_{\mathrm{m}} &= 0.32 + \frac{1.80}{h_{1/3}+0.6}, \\ \sigma &= \begin{cases} 0.08 & \text{if } \omega < \omega_{\mathrm{m}}, \\ 0.10 & \text{if } \omega > \omega_{\mathrm{m}}. \end{cases} \end{aligned}\right\} \tag{3.10}$$

The units in these formulae are m for $h_{1/3}$ and rad s^{-1} for ω.

4. Time integration of the roll equation

In linear roll analysis, which is valid for small roll amplitudes, spectral analysis is applied (Price & Bishop 1974; Lloyd 1989; SNAME 1989). The wave slope energy spectrum defined in the wave frequency domain has to be transformed into the encounter frequency domain. The same is done with the appropriate roll transfer function. Then, the roll energy spectrum is obtained by multiplying each spectral ordinate by the square of the roll transfer function. Finally, the roll energy spectrum may be transformed from the encounter frequency domain to the wave frequency domain. In this procedure, the grouping of the wave slope spectrum in the encounter frequency domain is obvious in the case of multi-valued encounter frequency (see figure 3). If the excitation energy is concentrated close to the vessel's roll natural frequency, large roll amplitudes occur and the capsizing of the vessel is possible.

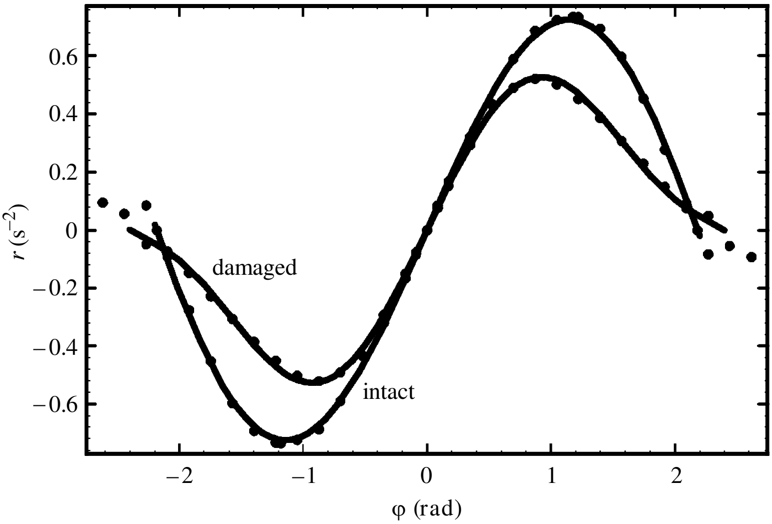

Figure 6. Relative restoring moment for the intact and damaged vessel (dotted line represents real, solid line approximate).

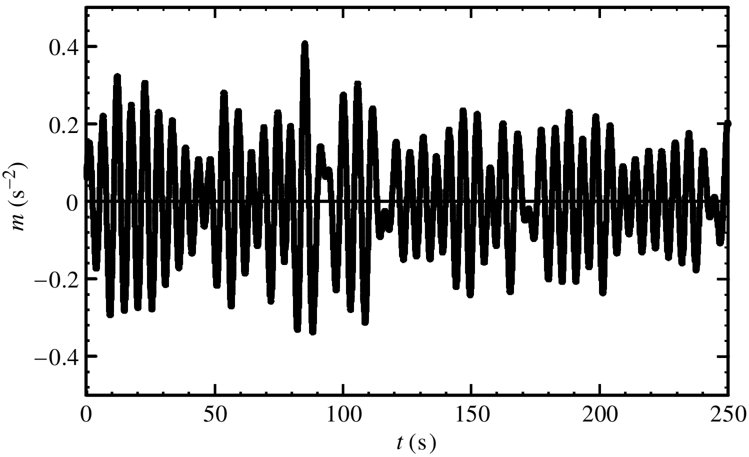

Figure 7. Relative excitation moment, $h_{1/3} = 2.5$ m, $U = 4$ m s^{-1}, $\chi = 60°$.

A vessel rolling with large amplitudes is a nonlinear problem and has to be analysed by nonlinear dynamics. Time-domain calculation is preferable, since only in that way may a complete response be obtained, especially for a random excitation.

The governing differential equation of rolling motion (2.5) is integrated by the harmonic acceleration method (Senjanović & Lozina 1993). For this purpose, the equation is transformed into a pseudo-linear form,

$$\ddot{\varphi} + 2\xi\omega_c\dot{\varphi} + \omega_c^2\varphi = \Psi(t), \tag{4.1}$$

where

$$\Psi(t) = m(t) + (2\xi\omega_c - d_1)\dot{\varphi} - d_3\dot{\varphi}^3 + (\omega_c^2 - k_1)\varphi - k_3\varphi^3 - k_5\varphi^5 - k_7\varphi^7 \tag{4.2}$$

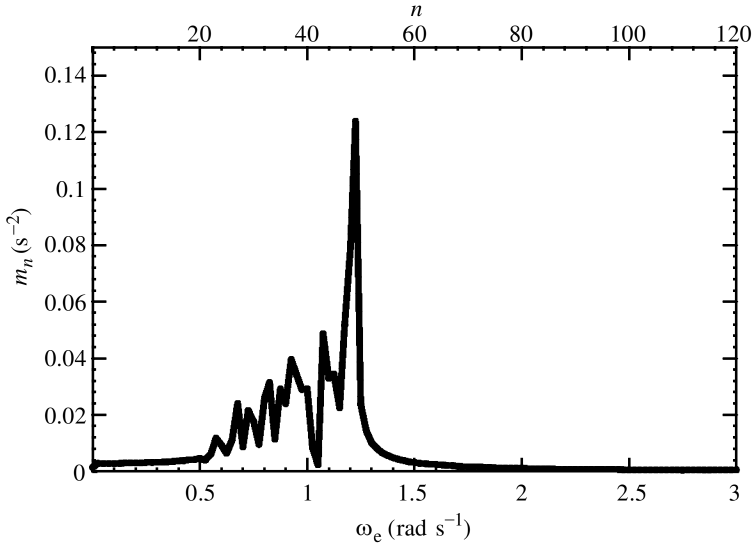

Figure 8. Fourier coefficients of relative excitation moment, $h_{1/3} = 2.5$ m, $U = 4$ m s^{-1}, $\chi = 60°$.

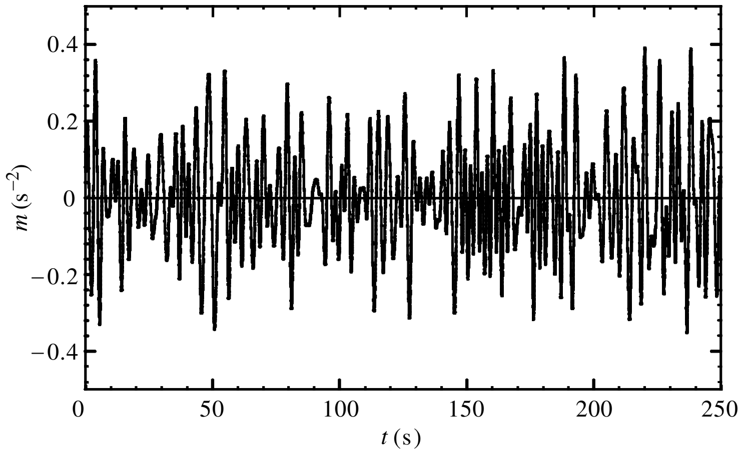

Figure 9. Relative excitation moment, $h_{1/3} = 2.5$ m, $\chi = 90°$.

is the pseudo-excitation, and ξ and ω_c are the assumed integration damping coefficient and integration frequency, respectively. These parameters are introduced in order to improve convergence (Senjanović & Lozina 1993). The step-by-step iteration integration algorithm reads as

$$\{Y\}_{i+1}^{k+1} = [T]\{Y\}_i + \{L\}\Psi_{i+1}^k, \tag{4.3}$$

where Y is the response vector, which includes φ, $\dot{\varphi}$ and $\ddot{\varphi}$, $[T]$ is the transfer matrix, $\{L\}$ is the load vector, i and k are the time and iteration steps, respectively. The quantities $[T]$ and $\{L\}$ are specified in Senjanović & Lozina (1993).

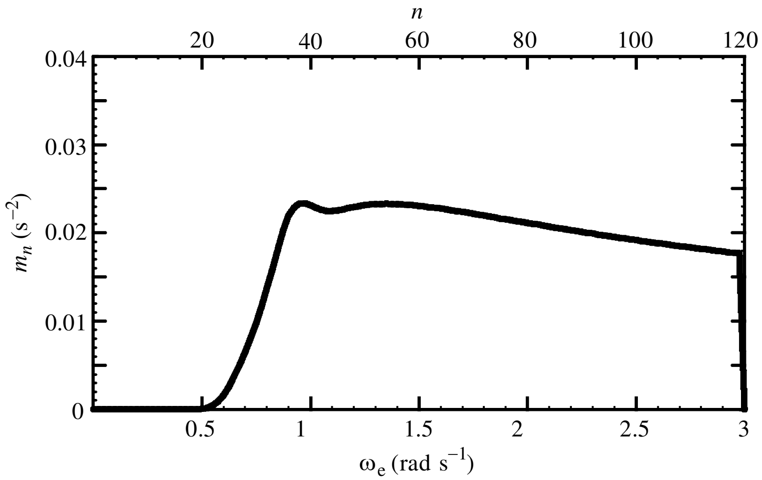

Figure 10. Fourier coefficients of relative excitation moment, $h_{1/3} = 2.5$ m, $\chi = 90°$.

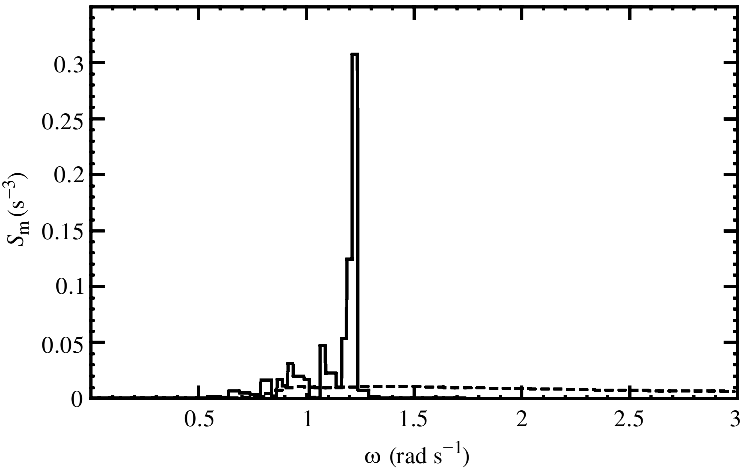

Figure 11. Energy spectrum of relative excitation moment, $h_{1/3} = 2.5$ m, $U = 4$ m s^{-1} (solid line represents $\chi = 60°$, dashed line $\chi = 90°$).

5. Vessel particulars and roll parameters

Rolling motion and survival analysis are illustrated in the case of a fishing vessel with the following particulars (Cardo *et al*. 1994):

length overall	$L_{oa} = 30.70$ m,
length between perpendiculars	$L_{pp} = 25.00$ m,
breadth	$B = 6.90$ m,
depth	$H = 4.96$ m,
draught	$T = 2.67$ m,
displacement	$\Delta = 195$ t.

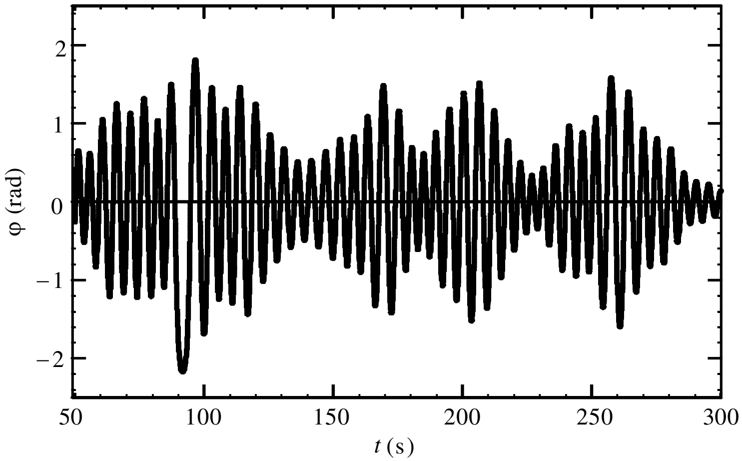

Figure 12. Roll angle of the intact vessel, $h_{1/3} = 2.5$ m, $U = 4$ m s^{-1}, $\chi = 60°$.

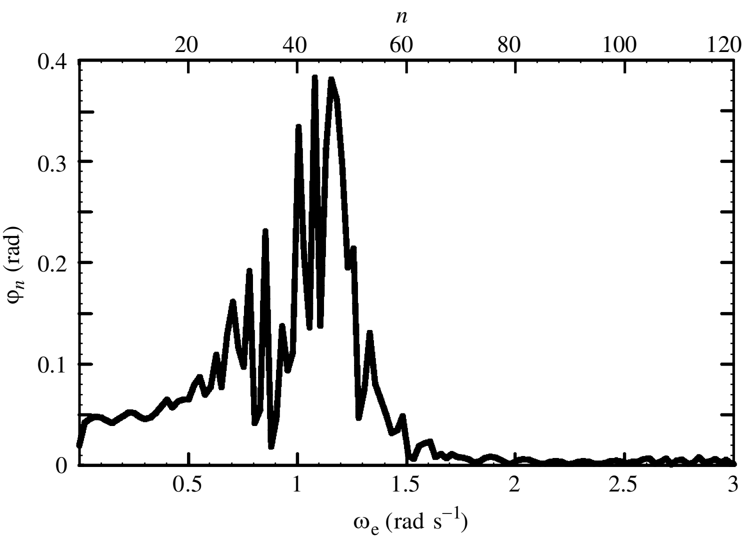

Figure 13. Fourier coefficients of the intact vessel's roll angle, $h_{1/3} = 2.5$ m, $U = 4$ m s^{-1}, $\chi = 60°$.

The roll parameters are determined by the model test for a model on a scale of 1:12.5. Most of the results are taken from Cardo *et al*. (1994) and recalculated for the full scale. The virtual mass moment of inertia $I = 1078$ t m^{-2}. The initial metacentric height $\overline{GM} = 0.962$ m. The roll natural frequency

$$\omega_o = \left[\frac{g\Delta\overline{GM}}{I}\right]^{1/2} = 1.32 \text{ rad s}^{-1}. \tag{5.1}$$

For the given vessel form shown in figure 5 and for the vertical coordinate of the centre of gravity, $\overline{KG} = 2.62$ m.

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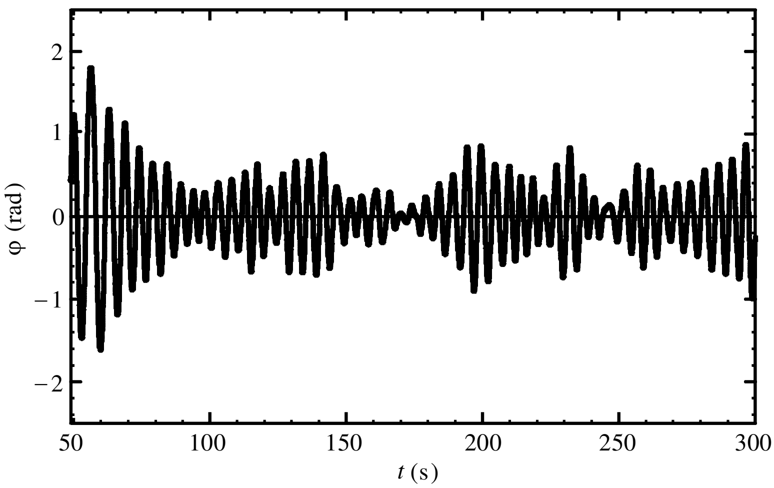

Figure 14. Roll angle of the intact vessel, $h_{1/3} = 2.5$ m, $\chi = 90°$.

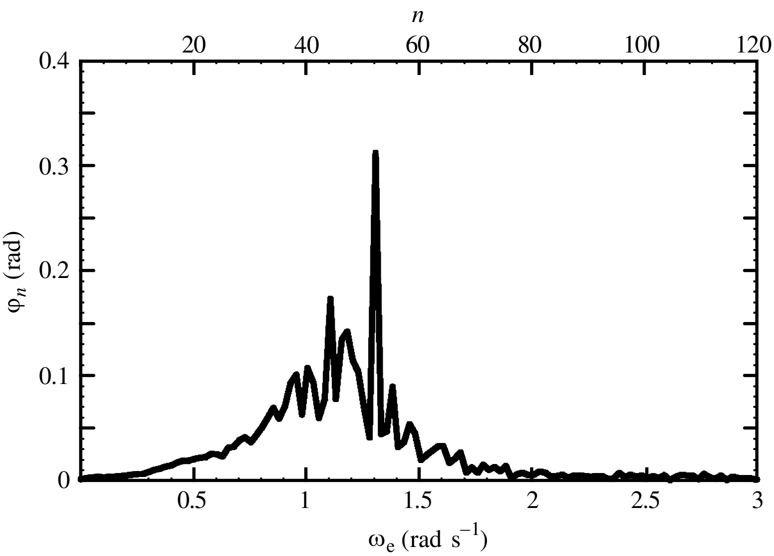

Figure 15. Fourier coefficients of the intact vessel's roll angle, $h_{1/3} = 2.5$ m, $\chi = 90°$.

The relative restoring moment reads as

$$r(\varphi) = \frac{R(\varphi)}{I} = \frac{g\Delta\overline{GZ}}{I}, \tag{5.2}$$

where $\overline{GZ}$ is the righting arm. The moment is shown by the dotted lines in figure 6 for the intact and damaged vessel. It is assumed that the engine room is flooded and the permeability coefficient is 0.75 (figure 1). Furthermore, the moment is approximated by a polynomial (solid lines in figure 6). The coefficients of the fifth- and seventh-order polynomial for the intact and damaged vessel, respectively, are

$$k_1 = 1.7737\ \mathrm{s}^{-2}, \qquad k_3 = -0.518\,291\ \mathrm{s}^{-2}, \qquad k_5 = 0.030\,680\,8\ \mathrm{s}^{-2}$$

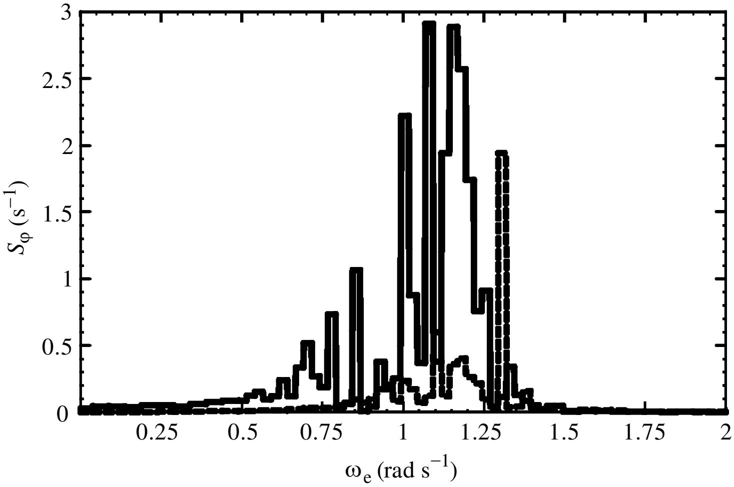

Figure 16. Roll energy spectra of the intact vessel, $h_{1/3} = 2.5$ m, $U = 4$ m s^{-1} (solid line represents $\chi = 60°$, dashed line $\chi = 90°$).

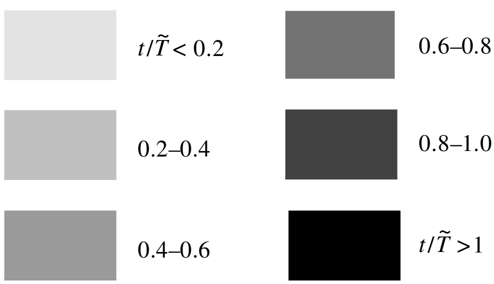

Figure 17. Legend of time-intervals to capsize.

and

$$k_1 = 1.638\,24\ \mathrm{s}^{-2}, \qquad k_3 = -0.829\,256\ \mathrm{s}^{-2},$$
$$k_5 = 0.147\,76\ \mathrm{s}^{-2}, \qquad k_7 = -0.009\,231\,74\ \mathrm{s}^{-2}.$$

The damping-moment coefficients are assumed to be the same in both cases, i.e. for the intact and damaged vessel

$$d_1 = 0.0208\ \mathrm{s}^{-1}, \qquad d_3 = 0.016\,48\ \mathrm{s}.$$

The effective wave slope coefficient $\alpha_\mathrm{o} = 0.729$ is taken from Cardo *et al.* (1994) for all harmonic wave components.

The vessel's roll analysis is performed for one sea state given by the significant wave height $h_{1/3} = 2.5$ m, three vessel speeds $U = 0$, 2, 4 m s^{-1}, and the heading angle $0° \leqslant \chi \leqslant 180°$, with discrete intervals $\Delta\chi = 10°$. The wave energy spectrum, shown in figure 4, up to the frequency $\omega_1 = 3$ rad s^{-1} is taken into account. This is the maximum value obtained by the formula

$$\omega_\mathrm{l} = \left[\frac{2\pi g \sin\chi}{B}\right]^{1/2}, \tag{5.3}$$

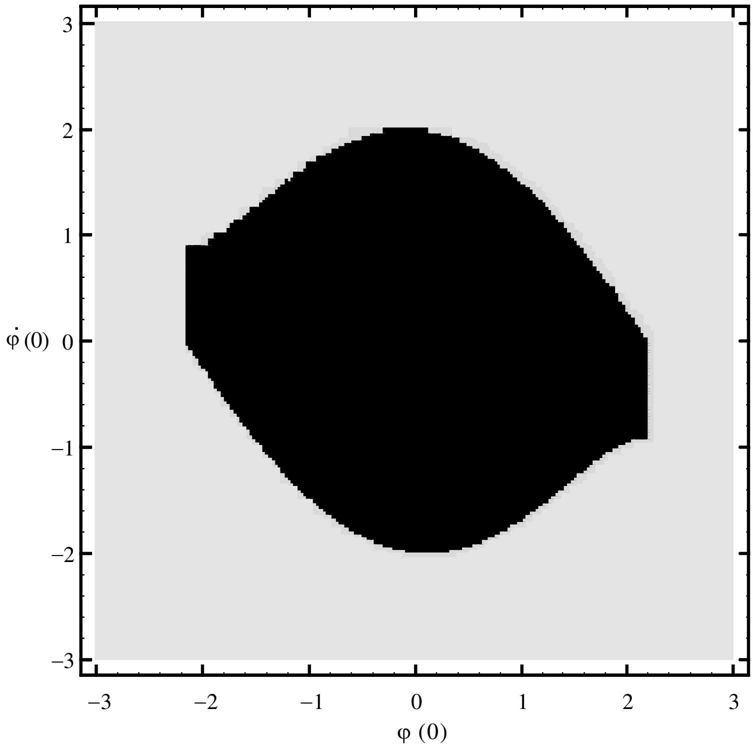

Figure 18. Safe basin of the intact vessel.

which follows from the assumed condition $\lambda \sin\chi > B$ for the wave slope excitation contribution and from the relation (3.7). The frequency step $\tilde{\omega} = 0.025\ \mathrm{rad\,s^{-1}}$ is chosen, so that the total number of harmonic waves forming an irregular wave pattern is $N = \omega_{\mathrm{l}}/\tilde{\omega} = 120$.

The relative excitation moment is calculated by the formula (3.5), employing (3.6), (3.7) and (3.8), within the time-interval equal to the period of the lowest excitation harmonic, i.e. $\tilde{T} = 2\pi/\tilde{\omega} = 250$ s. Also, the Fourier transform is performed in the encounter frequency domain. Interesting results for $h_{1/3} = 2.5$ m, $U = 4\ \mathrm{m\,s^{-1}}$ and $\chi = 60$ and $90°$ are obtained and shown in figures 7–10. The grouping of the harmonic waves is evident in the case $\chi = 60°$ at $\omega_{\mathrm{e}} = 1.2\ \mathrm{rad\,s^{-1}}$ (see figure 8), since, according to figure 2, their uniformly distributed frequencies are transformed into the encounter frequencies of almost equal values. The Fourier coefficients in the case $\chi = 90°$ follow a smooth line because $\omega_{\mathrm{e}} = \omega$, and their total number is equal to the number of the wave energy ordinates, i.e. $N = 120$ (see figure 10).

In order to compare the wave excitations for $\chi = 60$ and $90°$, the energy spectrum of the relative excitation moment is determined by the formula,

$$S_{\mathrm{m}}(\omega_{\mathrm{e}}) = \frac{m_n^2}{2\tilde{\omega}}, \tag{5.4}$$

and shown in figure 11. The grouping of component waves energy is very pronounced in the former case.

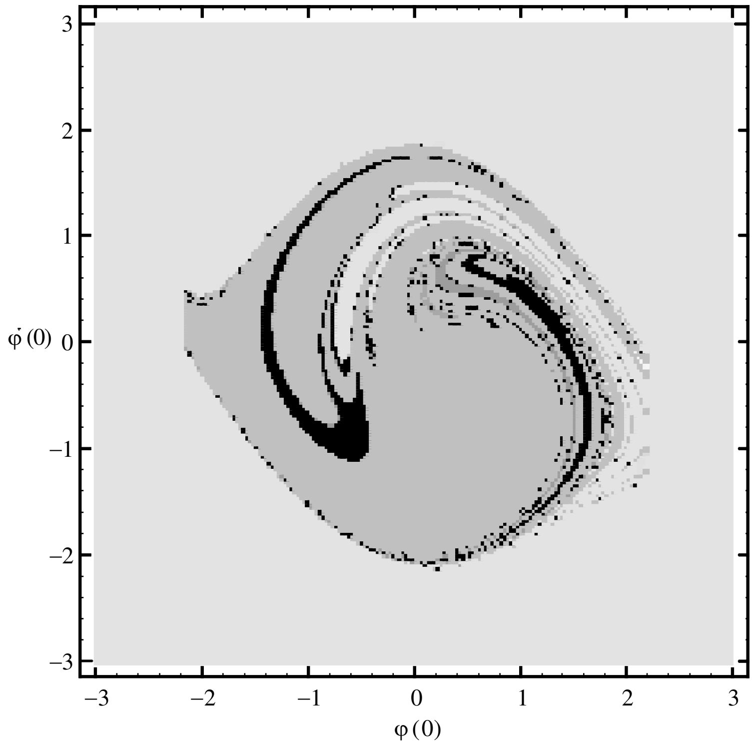

Figure 19. Capsize boundary of the intact vessel, $h_{1/3} = 2.5$ m, $U = 4$ m s^{-1}, $\chi = 60°$.

6. Roll analysis of intact vessel

The differential equation of rolling motion (2.5) is integrated by the harmonic acceleration method, taking the integration frequency close to the excitation spectrum peak frequency for beam seas, i.e. $\omega_c = 1$ rad s^{-1}, and damping coefficient $\xi = d_1/(2\omega_o) = 0.007\,88$. The initial conditions $\varphi(0) = \varphi_0$ and $\dot{\varphi}(0) = \dot{\varphi}_0$ are assumed so that the vessel stability is preserved within the time-interval $t = 300$ s, where the time-step is $\Delta t = 0.025$ s. This includes 50 s to reduce possible transient response caused by initial conditions, and the period of the lowest excitation harmonic $\tilde{T} = 2\pi/\tilde{\omega} = 250$ s.

Among a large number of the calculated cases for one sea state, three vessel speeds and 19 heading angles (as stated in § 5), the realization of the roll angle and its Fourier transform for $h_{1/3} = 2.5$ m, $U = 4$ m s^{-1} and $\chi = 60$ and 90° are shown in figures 12–15. Comparing these diagrams with those for the wave excitation (figures 8 and 10), some subharmonic response may be noticed. Also, stationary response is not achieved in a deterministic sense.

To be able to compare the vessel's response obtained for various conditions, the roll energy spectrum is determined according to the definition

$$S_\varphi(\omega_e) = \varphi_n^2/2\tilde{\omega}. \tag{6.1}$$

The roll spectrum for $h_{1/3} = 2.5$ m, $U = 4$ m s^{-1} and $\chi = 60$ and 90° is shown in figure 16. Roll energy, presented by a zero statistical moment, is much larger in the former than in the latter case.

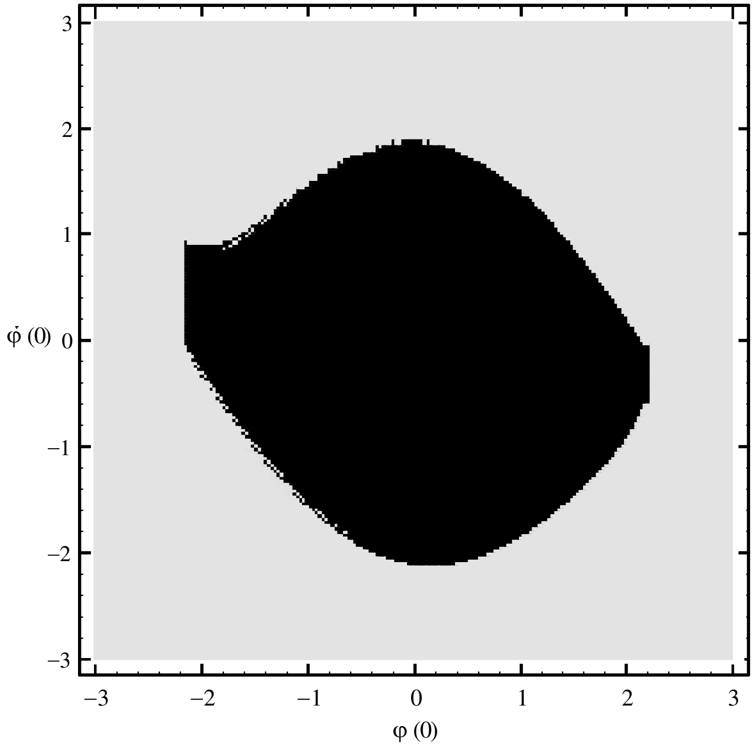

Figure 20. Capsize boundary of the intact vessel, $h_{1/3} = 2.5$ m, $\chi = 90°$.

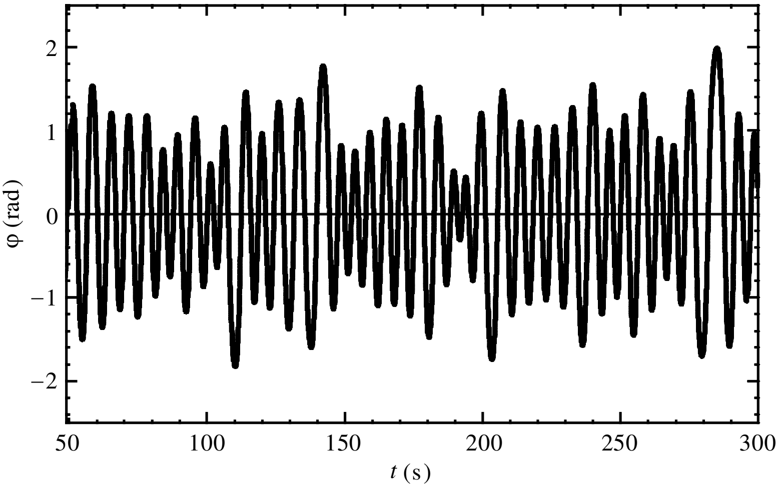

Figure 21. Roll angle of the damaged vessel, $h_{1/3} = 2.5$ m, $U = 4$ m s^{-1}, $\chi = 56°$.

The safety of the vessel rolling in rough seas may be analysed in the initial-value plane as a control space. For this purpose, the differential equation of motion (2.5) is integrated for a different combination of the initial conditions $\varphi(0) = \varphi_0$ and $\dot{\varphi}(0) = \dot{\varphi}_0$, where $-3 \leqslant \varphi_0 \leqslant 3$ rad and $-3 \leqslant \dot{\varphi}_0 \leqslant 3$ rad s^{-1}. Also, two sea states

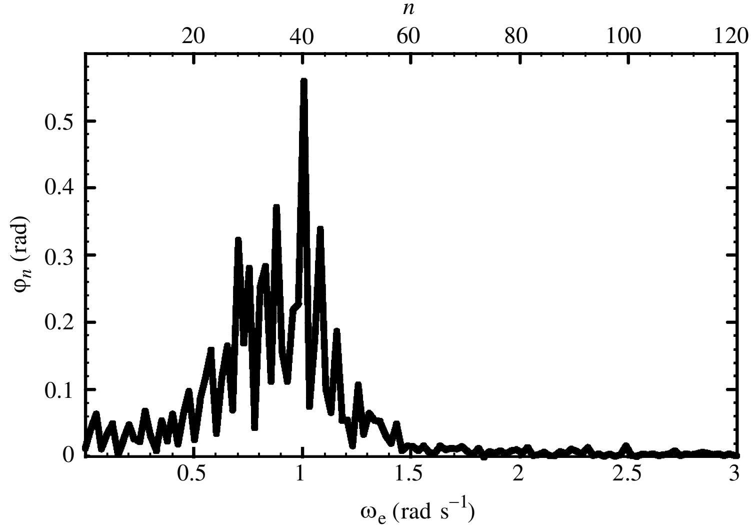

Figure 22. Fourier coefficients of the damaged vessel's roll angle, $h_{1/3} = 2.5$ m, $U = 4$ m s^{-1}, $\chi = 56°$.

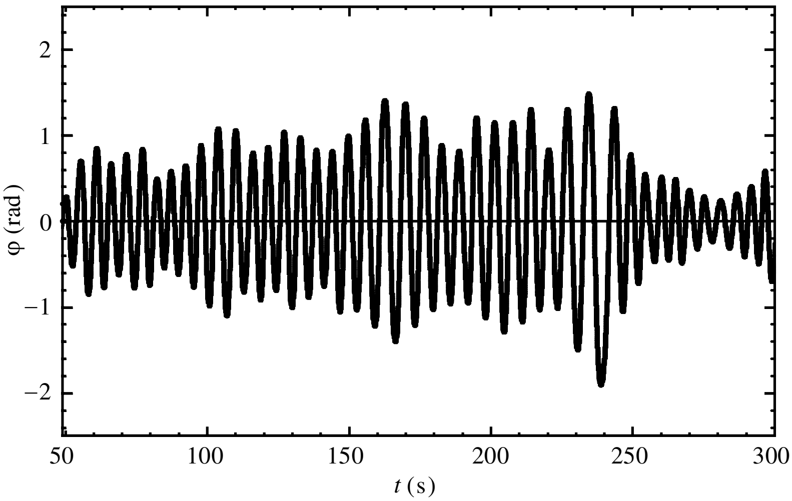

Figure 23. Roll angle of the damaged vessel, $h_{1/3} = 2.5$ m, $\chi = 90°$.

given by the significant wave height $h_{1/3} = 0$ and 2.5 m, three vessel speeds $U = 0$, 2 and 4 m s^{-1}, and 19 values of heading angles $0 \leqslant \chi \leqslant 180°$, with discrete heading intervals $\Delta\chi = 10°$, are taken into account.

Some interesting results are shown in figures 18–20, with a resolution of $200 \times 200 = 40\,000$ grid points. According to the legend given in figure 17, the black area is the safe basin, since the vessel survives within the whole period $\tilde{T}$. The bright area represents the capsize domain where the vessel capsizes at the very beginning of rolling motion, $0 < t/\tilde{T} < \frac{1}{5}$. The remaining shades denote some transition time-intervals to capsize. In the case of calm sea ($h_{1/3} = 0$ m), the safe basin is compact with a smooth capsize boundary (see figure 18). For rough seas, the erosion of the safe basin is progressive

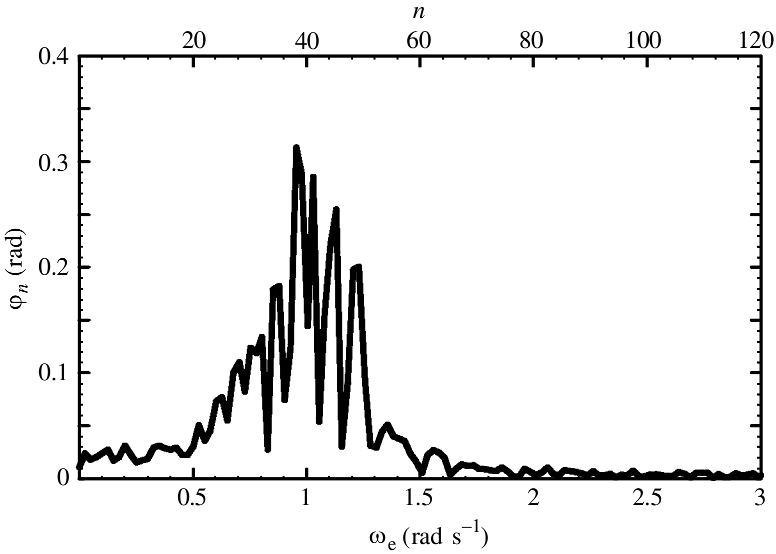

Figure 24. Fourier coefficients of the damaged vessel's roll angle, $h_{1/3} = 2.5$ m, $\chi = 90°$.

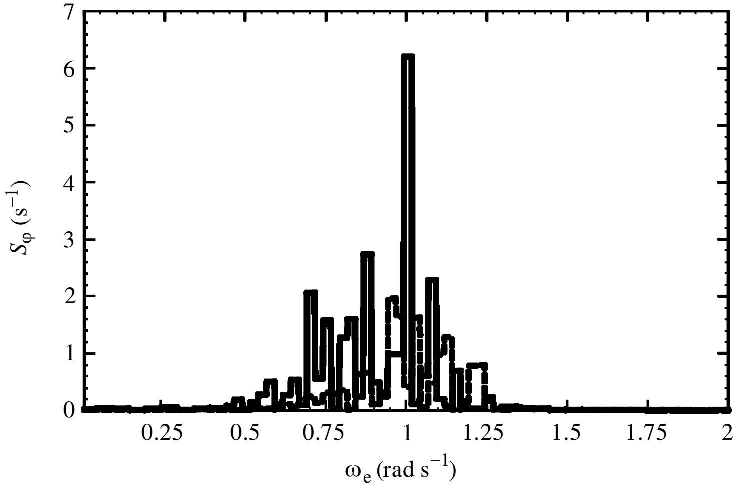

Figure 25. Roll energy spectra of the damaged vessel, $h_{1/3} = 2.5$ m, $U = 4$ m s^{-1} (solid line represents $\chi = 56°$, dashed line $\chi = 90°$).

with the sea roughness, as may be seen in Senjanović *et al.* (1996). It also depends on vessel speed and heading angle. In the case considered, where $h_{1/3} = 2.5$ m and $U = 4$ m s^{-1}, the erosion at $\chi = 90°$ is very small, while at $\chi = 60°$ it is at a maximum. This is due to the influence of the encounter frequency on the grouping of the harmonic component waves close to the natural frequency of vessel roll.

7. Roll analysis of damaged vessel

In a similar way, the rolling of the damaged vessel for the same sea state, vessel speeds and heading angles are analysed. The change of the excitation moment (3.5) due to

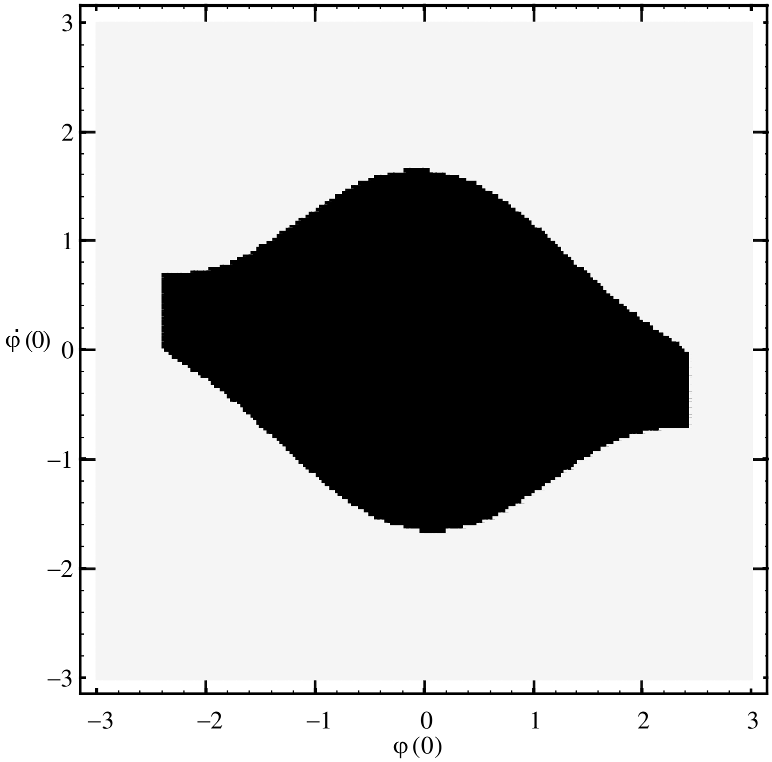

Figure 26. Safe basin of the damaged vessel.

Table 1. *Minimum probability of a vessel's survival, sea state* $h_{1/3} = 2.5$ m

vessel	U (m s^{-1})	χ (deg)	p
intact	0	90	0.99
	2	60	0.79
	4	60	0.10
damaged	0	90	0.98
	2	50	0.86
	4	56	0.16

different values of roll natural frequency ω_o is not taken into account. In order to find the maximum roll response, the heading angle in an interval $50 < \chi < 60°$ is varied by a step of $\Delta\chi = 1°$. For $h_{1/3} = 2.5$ m and $U = 4$ m s^{-1}, large rolling is obtained at $\chi = 56°$, while at $\chi = 90°$ the roll angle is somewhat lower, as may be seen from the time and frequency domain presentation of the realization in figures 21–24. The corresponding roll energy spectra are shown in figure 25. The zero statistical moment of the roll spectrum for $\chi = 56°$ is much larger than that for $\chi = 90°$. Comparing the diagrams in figures 16 and 25 for the intact and damaged vessel, respectively, it is evident that the corresponding statistical moments are larger in the latter than in the former case.

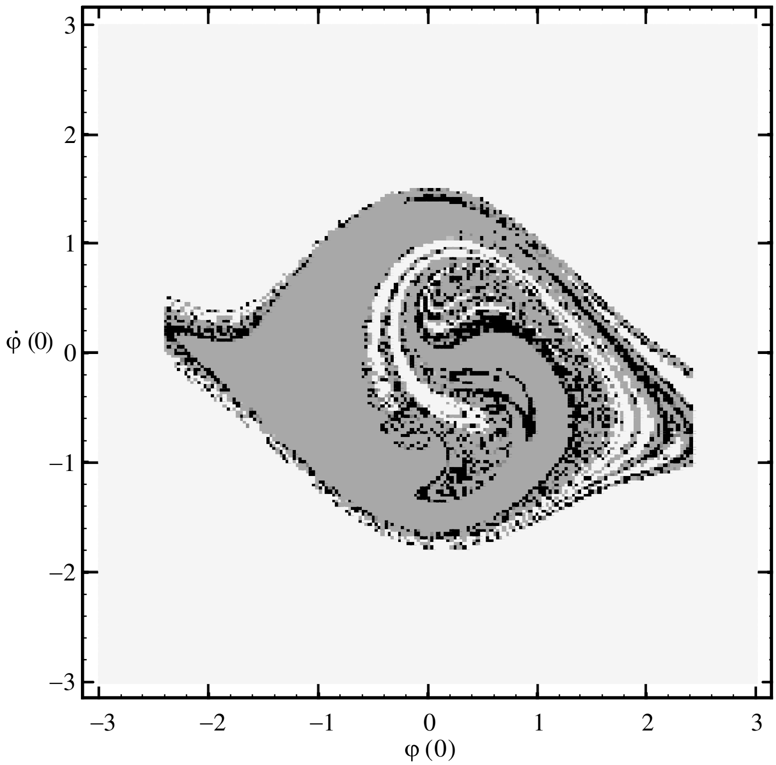

Figure 27. Capsize boundary of the damaged vessel, $h_{1/3} = 2.5$ m, $U = 4$ m s^{-1}, $\chi = 56°$.

Furthermore, the safe basin for calm sea (with $h_{1/3} = 0$ m) and its erosion for $h_{1/3} = 2.5$ m, $U = 4$ m s^{-1} and $\chi = 56$ and 90° are determined and shown in figures 26, 27 and 28, respectively. Comparing these figures for the damaged vessel with the corresponding figures 18–20 for the intact vessel, one may see that the safe basin is reduced by *ca.* 17% and more eroded in the former case, which means that the stability of the damaged vessel is decreased.

8. Probability of survival

The ratio between the areas of the safe basin obtained for and without wave excitation may be used as the probability of the vessel's survival (Senjanović *et al.* 1996, 1997). Such diagrams constructed for the intact and damaged vessel, and for the considered sea state $h_{1/3} = 2.5$ m, are shown in a PC radar map in figures 29 and 30. The vessel speed $U = 0$, 2 and 4 m s^{-1} is a parameter, and the heading angle is variable. The minimum values of the probability function for the considered cases are listed in table 1. The probability of survival of the floating vessel, $U = 0$ m s^{-1}, is minimum for beam seas, and it is larger for the intact than for the damaged condition. Quartering seas are very dangerous for the stability of a voyaging vessel. In spite of the fact that the probability of survival is somewhat larger for the damaged vessel than for the intact one, it is so drastically reduced in both cases that such situations have to be avoided. It is obvious that the vessel at quartering seas has to reduce its speed and/or change direction in order to avoid capsizing.

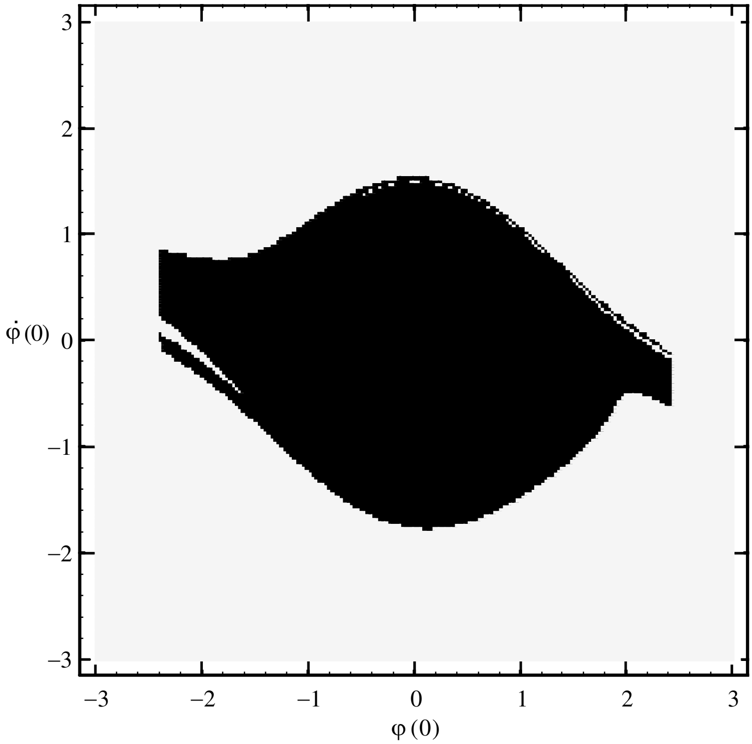

Figure 28. Capsize boundary of the damaged vessel, $h_{1/3} = 2.5$ m, $\chi = 90°$.

9. Conclusion

A vessel at sea is an autonomous system exposed to many hazardous situations. However, the vessel's capsizing has catastrophic consequences since it results in the loss of the vessel and, possibly, heavy human losses. Insufficient stability is a very frequent cause of fishing vessel losses (Dahle & Nisja 1984). The reason is that stability criteria, given by national and international rules, are rather conservative, prescribing the form and area under the restoring moment curve. In this way it is ensured that the work of the restoring moment in calm sea is larger than the work of a possible heeling moment. Many vessels capsize in rough seas in spite of the fact that they satisfy stability criteria. This is due to the fact that the nonlinear phenomena of vessels rolling are not taken into account. Therefore, in order to improve vessel safety, it is necessary to analyse vessel rolling in rough seas as a nonlinear random process.

In this paper a contemporary approach employing a sophisticated numerical method and recent knowledge of nonlinear dynamics is described. The problem is reduced to an SDOF system with a nonlinear damping and restoring moment. The wave energy spectrum is used to determine random roll excitation. An intact vessel and a damaged vessel are considered in different conditions specified by sea state, vessel speed and heading angle. The final result is the determination of the probability function of vessel survival depending on these three parameters. It is confirmed that voyage in quartering seas is very dangerous for vessel stability.

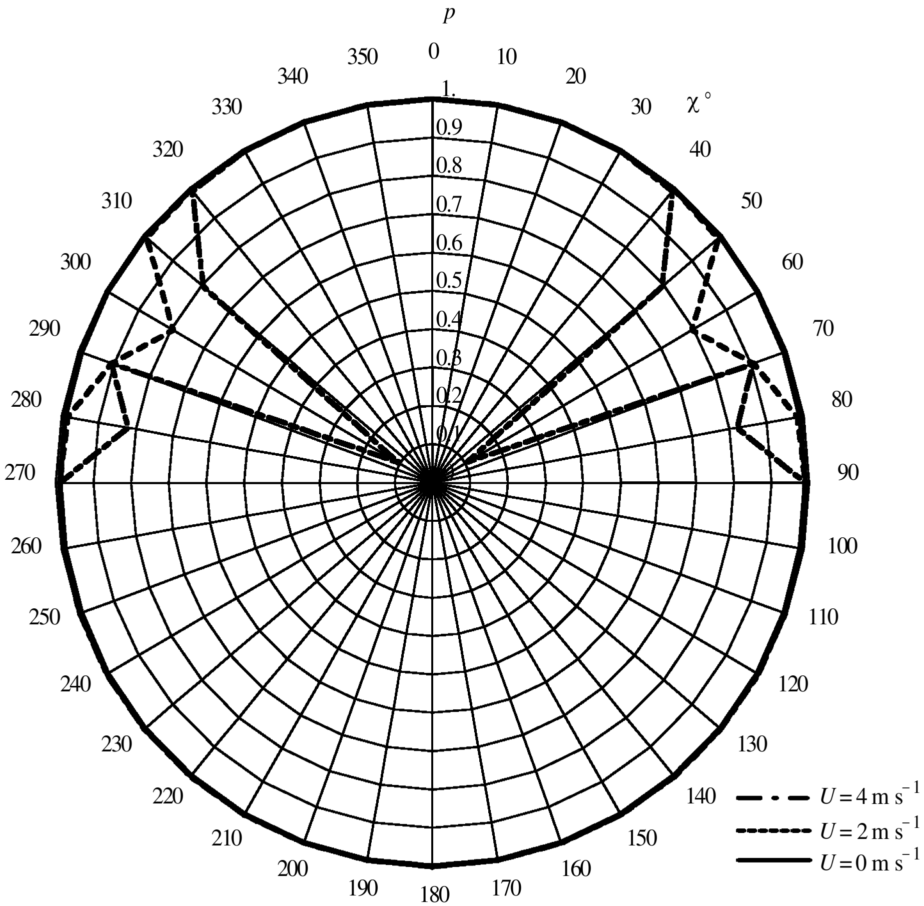

Figure 29. Probability of the intact vessel's survival, $h_{1/3} = 2.5$ m.

The intention of this paper is to point out the need for analysing vessel stability through nonlinear rolling and capsizing. This new approach requires application of the stability criterion based on the probability function of vessel survival. In this way, non-permissible voyage conditions may be specified. In the meantime, such survival diagrams like those shown in figures 29 and 30 may be used onboard to avoid disastrous situations. At the present time, this is left to the captain's assessment and experience.

In order to increase vessel safety in rough seas, this new approach for stability estimation should be improved by investigating coupled motion, hydrodynamic coefficients, reliable wave excitation, sloshing effect for damaged vessels, etc. (ITTC 1996; Contento *et al*. 1996*b*; Francescutto & Contento 1997). Finally, experimental verification is also very important (Oh *et al*. 1992; Francescutto & Contento 1994).

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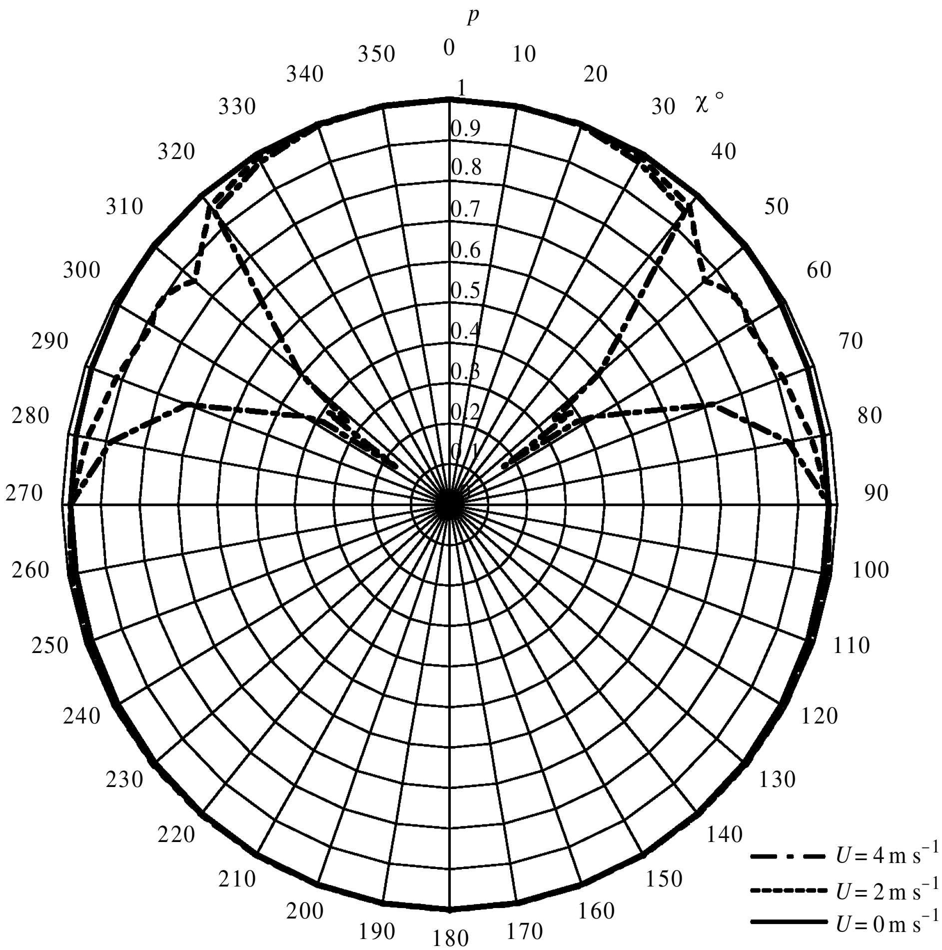

Figure 30. Probability of the damaged vessel's survival, $h_{1/3} = 2.5$ m.

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